NUMERICAL REPRESENTATION OF THE LARGE SCALES OF TURBULENCE

R. D. Moser[a], **P. Zandonade**[b] and **P. Vedula**[c]

Department of Theoretical and Applied Mechanics
University of Illinois
104 S. Wright St.
Urbana, IL 61801
[a]r-moser@uiuc.edu, [b]zandonad@uiuc.edu, [c]pvedula@uiuc.edu

Large-eddy simulation (LES) of fluid turbulence is one of a class of multi-scale problems in which one seeks to describe only the largest scales of the solution. One is then concerned with the small scales only to the extent that they affect the large scales. Other examples include the continuum approximation, shock capturing, homogenization of complex materials (e.g. composites), climate modeling and cosmology. Turbulence, which is also part of the climate modeling problem, is unusual because the small scales arise from intrinsic chaotic dynamics, and because there is no natural separation between the large scales of interest and the small scales to be modeled in an LES. These attributes require that some care be taken in formulating an LES. In this talk, the formulation of LES, its relation to numerical discretization and a new approach to LES modeling will be explored.

A starting point for LES development is the definition of the large-scales to be simulated. This is similar to numerical discretization in that a finite-dimensional representation of an infinite dimensional solution is defined. However, in this case, there is no consideration of the convergence of the representation, since by definition the representation does not include all relevant scales. The mapping to the finite dimensional representation (the filter) can be defined as in common numerical representations such as Fourier spectral, finite volume and finite element representations. Clearly, the unrepresented small scales and the way they are modeled depend on the representation used. For specificity and simplicity, primarily finite volume representations will be discussed here, but similar considerations apply for other schemes.

In a standard finite volume numerical discretization of the Navier-Stokes equations, the evolution of the state variables (volume-averaged velocities) is written in terms of the surface momentum fluxes and forces, which are approximated from the state variables by assuming that the velocity fields are smooth on the scale of the volumes. In the LES context, this assumption is not valid as illustrated in figure 1. Furthermore, because the volume averaged velocities provide so little information about the surface fluxes, these fluxes can be considered to be stochastic. Determining the fluxes is the primary modeling problem. A natural choice for a deterministic model is the average flux, conditioned on the values of the state variables (the volume-averaged velocities). This model has the good properties that it minimizes the mean-square error and it guarantees that the statistics of the resulting LES will match those of filtered turbulence [1, 3]. We call this the ideal LES evolution.

Unfortunately, the ideal LES is a conditional average with a large number of conditions (the number of degrees of freedom in the representation) and is therefore impractical to determine. But, formally approximating the ideal LES evolution is an attractive approach to constructing an LES model. Stochastic estimation is such an approximation, which we use to develop optimal LES models. The inputs to optimal LES are the correlations of the surface fluxes with the volume averaged velocities, which have been determined from direct numerical simulations and from theoretical considerations. The resulting models, have been used to perform a number of large eddy simulations [4, 2, 5], with exceptionally good results (see figure 2).

Figure 1: Model turbulent signal (red) and finite volume projection (black). Note that the projected signal provides little information about the derivatives at the volume boundaries.

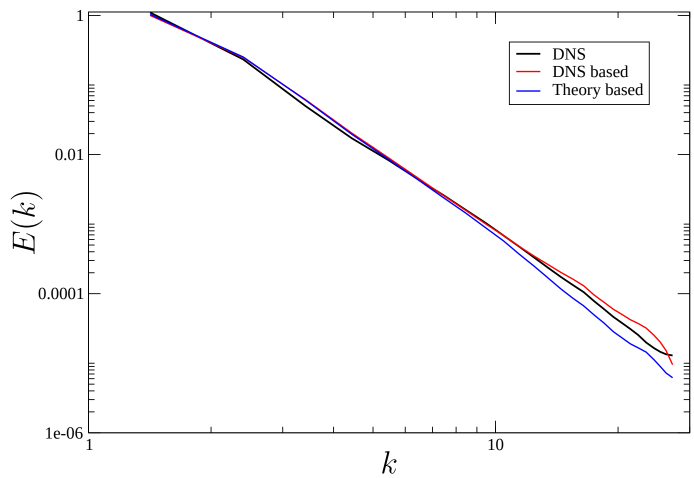

Figure 2: Spectrum for finite-volume filtered isotropic turbulence determined from both DNS and LES. DNS is at $Re_\lambda = 164$.

References

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